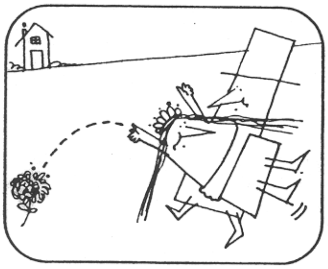
aha! Insight Martin Gardner

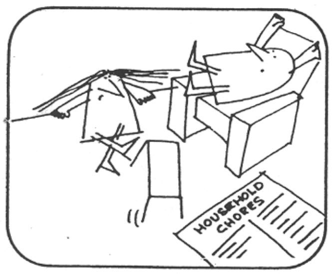
Scientific American, Inc./W.H.Freeman and Company
New York City

Dividing the Chores

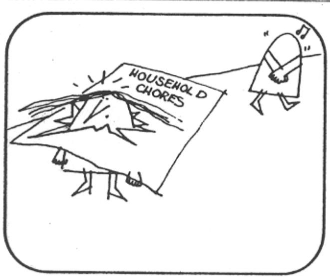


Mr. and Mrs. Buster Jones have just been married. Each has a steady job, so they have agreed to share the household chores.

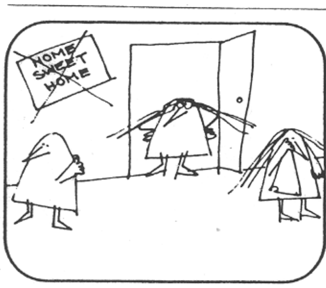
To divide the chores fairly, the Joneses made a list of all the jobs that had to be done in their apartment every week.
Buster: I've checked half the items, my love. Those are the chores for you to do.



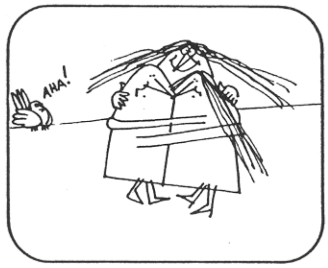
Janet: Sorry Buster, but I don't think you divided the list fairly. You've given me all the dirty jobs and you've taken all the easy ones.



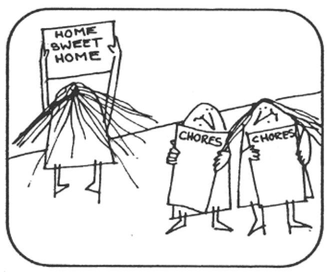
Then Mrs. Jones went over the list and marked the jobs she wanted to do. But Buster wouldn't agree.
Janet: If you expect me to do all these things you're out of your bird.



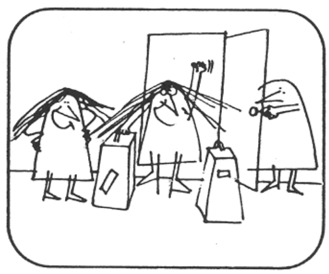
While they were still arguing the doorbell rang. It was Mrs. Jones' mother.
Mrs. Smith: What are you two love birds fighting about? I could hear you shouting as soon as I got off the elevator.



Mother listened while Buster and his wife explained the trouble. Suddenly she smiled and said.
Mrs. Smith: I've just thought of a marvelous solution. I'll show you how to divide the chores so both of you will be completely satisfied.



Mrs. Smith: One of you splits the list into two parts so that you would be willing to take either part. Then the other person gets to pick the half he or she wants first. Isn't that simple?



But it wasn't so simple a year later when Mother moved into the apartment. She agreed to take over one third of the chores but they couldn't decide how to divide the list fairly between the three of them. How do you suggest they do it?

Fair Division

The fair-division problem that has been answered is more usually given in terms of dividing a cake between two people so that each is satisfied he/she has at least half. The problem left unanswered is the same as that of dividing a cake fairly among three people so that each is satisfied he/she has at least a third.

The puzzle of fairly dividing a cake into thirds can be solved as follows. One person moves a large knife slowly over a cake. The cake may be any shape, but the knife must move so that the amount of cake on one side continuously increases from zero to the maximum amount. As soon as any one of the three believe that the knife is in a position to cut a first slice equal to ⅓ of the cake, he/she shouts "Cut!" The cut is made at that instant, and the person who shouted gets the piece. Since he/she is satisfied that he/she got ⅓, he/she drops out of the cutting ritual. In case two or all three shout "Cut!" simultaneously, the piece is given to any one of them.

The remaining two persons are, of course, satisfied that at least ⅔ of the cake remain. The problem is thus reduced to the previous case, and the cake can be fairly divided by having one person cut and the other choose.

This clearly generalizes to *n* persons. As the knife moves across the cake, the first person to shout "Cut!" gets the first slice (or it is given arbitrarily to one of the two or more who shout simultaneously). Then the procedure is repeated with the $n-1$ persons who remain. This continues until only two persons are left. The final portion of cake is divided as before, or, if you prefer, simply by continuing the procedure with the moving knife. The general solution is an excellent example of proving an algorithm by mathematical induction. It is easy to see how the algorithm can be applied to a list of household chores to be divided among *n* participants so that each person is satisfied he is getting his fair share.

John H. Conway, a Cambridge University mathematician, has investigated the fair-division problem when the satisfaction demanded by the participants is much stronger. Instead of a procedure that gives each person what he/she thinks is at least his/her fair share, is there a procedure which guarantees that each person is also convinced that no one else has a share greater than his/her own? If you think about it you will see that the algorithm given above does not provide this guarantee if there are three or more people. Conway and others have found solutions for this stronger version when there are three participants, but so far as we know the problem remains unsolved for four or more persons.