

**HOMEWORK #5**  
**(due Wednesday, October 11)**

*Read:* Bertsekas and Tsitsiklis, §§2.1-2.2.

1. In a game of bridge you are dealt a hand of 13 cards. The distribution of suits within your hand is important for bidding and play (but you can solve this problem without understanding the game).

(a) Show calculations to verify the following probabilities:

$$P(4\spadesuit, 3\heartsuit, 3\clubsuit, 3\diamondsuit) \approx 0.026 \quad \text{and} \quad P(4\spadesuit, 4\heartsuit, 3\clubsuit, 2\diamondsuit) \approx 0.018$$

(b) Suppose now that the suits are not specified; you are only concerned about the “shape” of your hand. Thus (4, 3, 3, 3) means you are dealt 4 cards in one suit, and 3 cards in each of the other three suits. Verify the following probabilities:

$$P(4, 3, 3, 3) \approx 0.11 \quad \text{and} \quad P(4, 4, 3, 2) \approx 0.22$$

2. A standard deck of 52 playing cards has been well shuffled.

(a) What is the PMF for the number of aces in a five-card poker hand dealt from this deck?

(b) Suppose that cards are dealt face up until the first ace appears. Let  $Y$  be the number of non-aces turned over before the first ace. Find a formula for  $P(Y = k)$ ,  $k = 0, 1, \dots, 48$ .

3. Consider the Tri-State Megabucks lottery as described in class. Show that the PMF for the largest of the seven numbers drawn (including the bonus ball) is given by:

$$p(k) = \binom{k-1}{6} / \binom{42}{7}, \quad k = 7, \dots, 42$$

4. People often naively question the randomness of lottery numbers because they think they see patterns, such as consecutive numbers. Consider picking a subset of  $k$  numbers at random from  $\{1, 2, \dots, n\}$ . To find how many subsets have no two consecutive numbers, note that there must at least one unselected number as a “spacer” between any two selected numbers. With  $k$  selections we need  $k-1$  such spacers. By ignoring the spacers, we can view our problem as an ordinary choice of  $k$  objects from  $n-(k-1)$ , putting the spaces back later. Thus, there is a 1-1 correspondence between subsets of size 3 from  $\{1,2,3,4,5,6\}$  with no two consecutive numbers, and the  $\binom{6-(3-1)}{3} = 4$  subsets of size 3 from  $\{1,2,3,4\}$ .

The table below illustrates the correspondence. Replacing each spacer inserts one number, shifting all elements on its right up by 1:  $1\ 1\ 2\ |3 \leftrightarrow 1\ 1\ 2,\ 4 \leftrightarrow 1,3,5$ .

Subset of $\{1,\dots,6\}$ , no consec	Subset of $\{1,\dots,4\}$ with spacers
$\{1,3,5\}$	1 1 2   3
$\{1,3,6\}$	1 1 2   4
$\{1,4,6\}$	1   3   4
$\{2,4,6\}$	2   3   4

- (a) In the six numbers drawn for the Megabucks jackpot (ignoring the bonus ball), what is the probability that there is at least one pair of consecutive numbers?
- (b) Shown below are the 78 drawings this year (through September), as reported on the New Hampshire Lottery web site. What fraction of these drawing had at least one pair of consecutive numbers (e.g., Sept 27 had 03-04 and Sept 20 had 11-12-13)?

Date	Jackpot Numbers	Date	Jackpot Numbers
09/30/2006	01-03-24-37-39-40	05/17/2006	11-18-22-25-35-40
09/27/2006	03-04-17-22-33-34	05/13/2006	11-13-23-24-26-36
09/23/2006	02-04-05-08-18-39	05/10/2006	03-16-17-18-19-29
09/20/2006	11-12-13-26-28-32	05/06/2006	07-11-13-17-19-41
09/16/2006	02-13-20-23-29-33	05/03/2006	16-19-26-34-35-42
09/13/2006	08-11-24-28-35-41	04/29/2006	03-16-18-25-32-40
09/09/2006	05-07-09-12-13-21	04/26/2006	05-09-10-11-19-26
09/06/2006	09-13-17-19-26-32	04/22/2006	02-13-14-18-35-39
09/02/2006	04-06-17-25-29-38	04/19/2006	05-18-20-22-27-31
08/30/2006	09-23-31-35-40-42	04/15/2006	03-06-10-21-27-31
08/26/2006	03-05-15-21-22-31	04/12/2006	09-11-19-29-35-37
08/23/2006	02-05-09-22-26-37	04/08/2006	08-23-29-30-34-41
08/19/2006	02-17-25-29-30-33	04/05/2006	06-17-21-34-35-39
08/16/2006	06-09-13-14-24-39	04/01/2006	12-13-30-32-39-42
08/12/2006	05-06-10-17-20-21	03/29/2006	13-15-17-19-29-31
08/09/2006	01-09-14-16-29-33	03/25/2006	13-22-24-29-36-39
08/05/2006	02-12-26-29-33-38	03/22/2006	01-16-17-18-26-33
08/02/2006	11-15-20-21-34-39	03/18/2006	02-14-15-21-37-42
07/29/2006	02-13-19-23-34-38	03/15/2006	12-13-14-24-29-42
07/26/2006	05-06-09-13-29-39	03/11/2006	11-15-24-25-28-36
07/22/2006	11-21-30-31-35-37	03/08/2006	18-21-22-24-39-42
07/19/2006	08-14-23-24-35-41	03/04/2006	01-03-19-25-31-32
07/15/2006	02-16-26-27-33-37	03/01/2006	03-13-29-34-39-41
07/12/2006	03-18-20-24-29-30	02/25/2006	08-15-20-24-34-37
07/08/2006	03-06-11-29-33-37	02/22/2006	04-11-13-14-16-37
07/05/2006	02-05-11-14-26-40	02/18/2006	06-09-11-12-17-23
07/01/2006	12-16-30-34-39-40	02/15/2006	04-05-19-22-32-38
06/28/2006	11-14-15-22-24-27	02/11/2006	03-15-33-34-35-38
06/24/2006	05-07-15-21-25-34	02/08/2006	23-25-28-29-31-35
06/21/2006	18-30-31-33-41-42	02/04/2006	15-29-31-35-38-40
06/17/2006	02-03-08-11-18-22	02/01/2006	08-09-15-31-37-39
06/14/2006	03-14-26-28-34-37	01/28/2006	02-14-15-22-37-38
06/10/2006	01-07-13-26-35-37	01/25/2006	03-10-22-26-29-40
06/07/2006	09-13-29-32-35-40	01/21/2006	02-05-25-30-34-36
06/03/2006	13-16-17-19-35-36	01/18/2006	14-25-27-30-36-39
05/31/2006	04-09-13-23-32-41	01/14/2006	03-07-26-30-31-36
05/27/2006	11-22-27-30-31-40	01/11/2006	04-10-13-29-31-35
05/24/2006	06-09-20-36-40-41	01/07/2006	11-17-21-24-28-35
05/20/2006	02-07-08-12-14-39	01/04/2006	04-06-10-18-25-33

Source: <http://www.nhlottery.org/winning-numbers/pwn-mb-2006.asp>